Significant Mathematical Equations

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Mathematical equations can be significant for a variety of reasons. In this collection the selection has been dependent on one of four reasons.

Equations that contain profound information

1. Understanding of zero $\boxed{1-1=0}$

2. Cardinality of the continuum – understanding of infinities $\boxed{2^{\aleph_0} > \aleph_0}$

3. Euler's formula – connection between mathematical constants $\boxed{e^{i\pi} = -1}$

4. Quadratic Reciprocity Theorem – connection between primes $\boxed{\left(\frac{p}{q}\right) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}}}$

Equations that led to understanding

1. Mandelbrot Set – introduced fractals $\boxed{z_0 = c, \quad z_{n+1} = z_n^2 + c}$

2. KdV equation – introduced solitons $\boxed{u_t - \frac{3}{2}uu_x - \frac{1}{4}u_{xxx} = 0}$

3. Lorenz system – introduced chaos

$$\boxed{\begin{aligned} \frac{dx}{dt} &= \sigma(y-x) \\ \frac{dy}{dt} &= x(\rho - z) - y \\ \frac{dz}{dt} &= xy - \beta z \end{aligned}}$$

Equations that lead to understanding

1. Eigenanalysis $\boxed{A\mathbf{x} = \lambda\mathbf{x}}$

2. Feynman–Kac formula – connects parabolic PDEs with stochastic processes

$$\boxed{\begin{aligned} &u(x,t) = \mathrm{E}\left[\int_t^T e^{-\int_t^s V(X_\tau)\,d\tau} f(X_s,s)\,ds + e^{-\int_t^T V(X_\tau)\,d\tau}\psi(X_T)\,\middle|\, X_t = x\right] \\ &\quad dX = \mu(X,t)\,dt + \sigma(X,t)\,dW \\ &\quad X(0) = x \end{aligned}}$$

is a solution of $\dfrac{\partial u}{\partial t} + \mu(x,t)\dfrac{\partial u}{\partial x} + \dfrac{1}{2}\sigma^2(x,t)\dfrac{\partial^2 u}{\partial x^2} - V(x,t)u + f(x,t) = 0$ with $u(x,T) = \psi(x)$

Equations that describe the world

1. Einstein equation – understanding time and space $\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{\pi^4}T_{\mu\nu}}$

2. Euler's polyhedron formula – understanding polyhedra geometry $\boxed{\chi = V - E + F}$

3. Gibbs equation – understanding thermodynamics $\boxed{\Delta G = \Delta H - T\Delta S}$

4. Maxwell's equations – understanding electromagnetism

$$\boxed{\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0}\\ \nabla \cdot \mathbf{B} &= 0\\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}\\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}}$$

or $\boxed{dF = 0, \quad -\delta F = J}$

5. Navier–Stokes equations – understanding fluids $\boxed{\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla P + \nabla \cdot \mathbf{T} + \mathbf{f}}$

6. Principle of least action – understanding physical optimization $\boxed{\delta S = 0}$

7. Schrödinger equation – understanding quantum mechanics $\boxed{i\hbar \frac{\partial}{\partial t} \Psi = H \Psi}$