# Errors in the 30th edition of Standard Mathematical Tables and Formulae (Second and later printings) <br> LAST UPDATED: February 25, 2002 

NOTES:

1. Due to our procedures for verifying errata, the date that an entry is updated may be significantly later than the date that the errata was brought to our attention.
2. Sometimes many contributors bring the same errata to our attention.
3. The .pdf and .ps versions of this errata list include an index of dates (see page ??) of when the errata were updated.

## ERRATA:

1. Preface, page vii, the editor-in-chief's email address is listed as zwilling@world.std.com. This is now outdated, the correct email address is zwillinger@alum.mit.edu.
This entry updated before 7 March 2001.
2. Preface, page vii, the following text should be added:

Errata for this book will be maintained at http://errata.mathtable.com This entry updated before 7 March 2001.
3. Section 1.1.2, REPRESENTATION OF NUMBERS, page 4, line 18 presently has $" 575_{10}=3 \mathrm{BA}_{12}$ ", which is incorrect. It should have been " $574_{10}=3 \mathrm{BA}_{12}$ ".
(Thanks to Richard Hughes for this correction.)
This entry last updated 25 Februrary 2002.
4. Section 1.1.4, ROMAN NUMERALS page 5:

- Rule number 2 is now:

A symbol following one of lesser value has the lesser value subtracted from the larger value (for example, $\mathrm{IV}=4, \mathrm{IX}=9, \mathrm{VM}=995$ ),
This is incorrect, it should have been
A symbol following one of lesser value has the lesser value subtracted from the larger value. An I is only allowed to precede a V or an X , an X is only allowed to precede an $L$ or a $C$, and a $C$ is only allowed to precede a $D$ or an $M$. (For example, $\mathrm{IV}=4, \mathrm{IX}=9, \mathrm{CD}=400$ ),
This entry updated before 7 March 2001.

- Rule number 3 is now, in part:

$$
(\text { for example, } \mathrm{XIV}=10+(5-1)=14, \text { CIX }=100+(10-1)=109, \mathrm{DVL}=
$$ $500+(50-5)=545)$.

This is incorrect, it should have been
$($ for example, $\mathrm{XIV}=10+(5-1)=14$, CIX $=100+(10-1)=109$, DXL $=$ $500+(50-10)=540)$.
(Thanks to Glebb Leider for these corrections.)
This entry updated before 7 March 2001.
5. Section 1.2.6, FACTORIALS, page 15, lines $7-8$, we presently have
$\ldots$ (also called the falling factorial ... (sometimes $a^{\underline{n}}$ ) ...
This is incorrect. It should have been
$\ldots$. (also called the rising factorial ... (sometimes $a^{\bar{n}}$ ) ...
[This brings the description into line with, e.g., D. Knuth, Art of Computer Programming, Vol I, 3rd Edition, Section 1.2.5, Permutations and Factorials, page 50.]
(Thanks to David Gehrig for this correction.)
This entry updated before 7 March 2001.
6. Section 1.2.6, FACTORIALS, page 15, the following text should be added

- Note that, since the empty product is 1 , it follows that $0!=1$.
- If $n$ is a negative integer $(n=-1,-2, \ldots)$ then $n!= \pm \infty$.
- The generalization of the factorial to non-integer arguments is the $\Gamma$ function (see page 494).
(Thanks to David Cantrell for these corrections.)
This entry updated before 7 March 2001.

7. Section 1.3.2, GENERAL PROPERTIES, page 31, item number 9, Holder's inequality is listed as

$$
\sum\left|a_{n} b_{n}\right| \leq \sum\left|a_{n}\right|^{1 / p} \sum\left|b_{n}\right|^{1 / q}
$$

This is incorrect. It should have been (note the exponents on the terms)

$$
\sum\left|a_{n} b_{n}\right| \leq\left(\sum\left|a_{n}^{p}\right|\right)^{1 / p}\left(\sum\left|b_{n}^{q}\right|\right)^{1 / q}
$$

(Thanks to Alain Boulanger for this correction.)
This entry updated before 7 March 2001.
8. Section 1.3.4, TYPES OF SERIES, page 35, Taylor series, item number 2, Lagranges's form: we presently have

$$
R_{N}=\frac{x^{N+1}}{(N+1)!} f^{(N+1)}(\theta a), \quad 0<\theta<1
$$

This is incorrect. It should have been

$$
R_{N}=\frac{x^{N+1}}{(N+1)!} f^{(N+1)}(a+\theta x), \quad 0<\theta<1
$$

(Thanks to Richard E. Stone for this correction.)
This entry updated before 7 March 2001.
9. Section 1.3.4, TYPES OF SERIES, page 36, Other types of series, item number 2, Arithmetic power series: the left hand side of the equation is shown as as

$$
\sum_{n=1}^{N}
$$

This is incorrect. It should have been (note the summation should start at 0 , not 1 )

$$
\sum_{n=0}^{N}
$$

(Thanks to Greg Iles for this correction.)
This entry updated before 7 March 2001.
10. Section 1.3.10, INFINITE SERIES, page 40, the following should be added:

$$
e^{x}=\frac{1}{1-x}+\sum_{n=1}^{\infty} \frac{x^{n+1}}{n!(x-n)(n+1-x)} \quad x \text { not a positive integer }
$$

(Thanks to Barry Pasternack for this addition.)
This entry updated before 7 March 2001.
11. Section 1.4.4, EXPANSIONS OF BASIC PERIODIC FUNCTIONS page 50,

- The fourth expansion is now

$$
f(x)=\frac{9}{\pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}} \sin \frac{n \pi \square a}{3} \sin \frac{n \pi x}{L} \quad\left(a=\frac{c}{2 L}\right)
$$

This is incorrect, it should have been

$$
f(x)=\frac{9}{\pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}} \sin \frac{n \pi}{3} \sin \frac{n \pi x}{L}
$$

- The fifth expansion is now

$$
f(x)=\frac{32}{3 \pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}} \sin \frac{n \pi \boxed{a}}{4} \sin \frac{n \pi x}{L} \quad\left(a=\frac{c}{2 L}\right)
$$

This is incorrect, it should have been

$$
f(x)=\frac{32}{3 \pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}} \sin \frac{n \pi}{4} \sin \frac{n \pi x}{L}
$$

This entry updated before 7 March 2001.
12. Section 1.5.15, TABLE OF TRANSFORMATIONS, page 57, presently has the following table entry (line 9)

$$
\operatorname{Re}(\cos (z))=\cos (x) \sinh (y), \quad \operatorname{Im}(\cos (z))=-\sin (x) \cosh (y)
$$

This is incorrect and should be:

$$
\operatorname{Re}(\cos (z))=\cos (x) \cosh (y), \quad \operatorname{Im}(\cos (z))=-\sin (x) \sinh (y)
$$

(Thanks to Kosa Gabor for this addition.)
This entry updated before 7 March 2001.
13. Section 1.6.5, METRIC SPACE, page 66, presently has the following line
positive definiteness: $\rho(x, y)>0$ for all $x, y$ in $E$, and $\rho(x, y)=0$ if and only if $x=y$.

This is incorrect and should be (note that " $>$ " is replaced with " $\geq$ "):
positive definiteness: $\rho(x, y) \geq 0$ for all $x, y$ in $E$, and $\rho(x, y)=0$ if and only if $x=y$.
(Thanks to Paul Stanford for this correction.)
This entry updated before 7 March 2001.
14. Section 1.7, GENERALIZED FUNCTIONS, page 72, item number 7 was

$$
\delta(x)=\frac{2}{L} \sum_{n=1}^{\infty} \sin \frac{n \pi \xi}{L} \sin \frac{n \pi x}{L} \text { for } 0<\xi<L \text { (Fourier sine series) }
$$

which is incorrect. It should have been

$$
\delta(x \boxed{-\xi})=\frac{2}{L} \sum_{n=1}^{\infty} \sin \frac{n \pi \xi}{L} \sin \frac{n \pi x}{L} \text { for } 0<\xi<L \text { (Fourier sine series) }
$$

This entry last updated 5 October 2001.
15. Section 1.7, GENERALIZED FUNCTIONS, page 72, the Heaviside function is defined as

$$
H(x)=\cdots= \begin{cases}0 & x<0 \\ 1 & x>1\end{cases}
$$

This is incorrect. It should have been

$$
H(x)=\cdots= \begin{cases}0 & x<0 \\ 1 & x>0\end{cases}
$$

(Thanks to Catherine Roberts for this correction.)
This entry updated before 7 March 2001.
16. Section 2.1.4, PARTIAL FRACTIONS, page 81,
(a) Repeated linear factor: The center of the page now has, in part

$$
\begin{align*}
A_{k} & =\frac{r(a)}{h(a)} \\
A_{k-1} & =\left.\frac{d}{d x}\left(\frac{r(x)}{g(x)}\right)\right|_{x=a} \\
A_{k-2} & =\left.\frac{1}{2!} \frac{d^{2}}{d x^{2}}\left(\frac{r(x)}{g(x)}\right)\right|_{x=a}  \tag{1}\\
A_{k-j} & =\left.\frac{1}{j!} \frac{d^{j}}{d x^{j}}\left(\frac{r(x)}{g(x)}\right)\right|_{x=a}
\end{align*}
$$

This is incorrect. It should have been (note that the $g(x)$ 's should have been $h(x)$ 's)

$$
\begin{align*}
A_{k} & =\frac{r(a)}{h(a)} \\
A_{k-1} & =\left.\frac{d}{d x}\left(\frac{r(x)}{h(x)}\right)\right|_{x=a} \\
A_{k-2} & =\left.\frac{1}{2!} \frac{d^{2}}{d x^{2}}\left(\frac{r(x)}{h(x)}\right)\right|_{x=a}  \tag{2}\\
A_{k-j} & =\left.\frac{1}{j!} \frac{d^{j}}{d x^{j}}\left(\frac{r(x)}{h(x)}\right)\right|_{x=a}
\end{align*}
$$

(b) Single quadratic factor: When $A$ and $B$ are both real, if after multiplying the equation by $g(x)$ a root of $x^{2}+b x+c$ is substituted for $x$, then the values of $A$ and $B$ can be inferred from this single complex equation by equating real and imaginary parts.
Similarly for Repeated quadratic factors. In other words, the number of equations needed is half the number stated.
(Thanks to David M. Bradley for these corrections.)
This entry updated before 7 March 2001.
17. Section 2.2.2, CUBIC POLYNOMIALS, page 82, the second paragraph is now

The solutions are given by $\sqrt[3]{\alpha}-\sqrt[3]{\beta}, e^{\frac{2 \pi i}{3}} \sqrt[3]{\alpha}-e^{\frac{4 \pi i}{3}} \sqrt[3]{\beta}$, and $e^{\frac{4 \pi i}{3}} \sqrt[3]{\alpha}-e^{\frac{2 \pi i}{3}} \sqrt[3]{\beta}$, where

This is incorrect. It should have been (note the signs)
The solutions are given by $\sqrt[3]{\alpha}+\sqrt[3]{\beta}, e^{\frac{2 \pi i}{3}} \sqrt[3]{\alpha}+e^{\frac{4 \pi i}{3}} \sqrt[3]{\beta}$, and $e^{\frac{4 \pi i}{3}} \sqrt[3]{\alpha}+e^{\frac{2 \pi i}{3}} \sqrt[3]{\beta}$, where

This entry updated before 7 March 2001.
18. Section 2.3.1, Values, page 87, presently has "There are 32 Carmichael numbers less than one million" which is incorrect, it should have been "There are 32 Carmichael numbers less than one half million".

This entry last updated 25 Februrary 2002.
19. Section 2.3.9, PRIME NUMBERS, page 94, line -5 , we presently have

$$
3^{6} \equiv 719 \equiv 16
$$

This is incorrect. It should have been

$$
3^{6} \equiv 729 \equiv 16
$$

(Thanks to See Chew for this correction.)
This entry updated before 7 March 2001.
20. Section 2.5.4, DETERMINANTS AND PERMANENTS, page 123, equation (2.5.10) presently is

$$
\left.|\mathbf{A}|=|\mathbf{E}| \mid \mathbf{B}-\mathbf{C E}^{-1} \mathbf{D}\right)|=|\mathbf{B}||\left(\mathbf{E}-\mathbf{D B}^{-1} \mathbf{C}\right) \mid
$$

There is a missing parenthesis, this should have been:

$$
|\mathbf{A}|=|\mathbf{E}|\left|\left(\mathbf{B}-\mathbf{C E}^{-1} \mathbf{D}\right)\right|=|\mathbf{B}|\left|\left(\mathbf{E}-\mathbf{D B}^{-1} \mathbf{C}\right)\right|
$$

(Thanks to Paul Stanford for this correction.)
This entry updated before 7 March 2001.
21. Section 2.5.6, SINGULARITY, RANK, AND INVERSES, page 125, item number 6,
(a) equation (2.5.16), the formula for $\mathbf{X}$ is given incorrectly. It now has

$$
\mathbf{X}=\left(\mathbf{B}-\mathrm{CE}^{-1}\right)^{-1}
$$

This is incorrect. It should have been (note the missing $D$ ):

$$
\mathbf{X}=\left(\mathbf{B}-\mathrm{CE}^{-1} \mathbf{D}\right)^{-1}
$$

(b) The following should be added after equation (2.5.17)

The inverse of a $2 \times 2$ matrix is as follows (defined when $a d \neq b c$ ):

$$
\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right]^{-1}=\frac{1}{a d-b c}\left[\begin{array}{cc}
d & -b \\
-c & a
\end{array}\right]
$$

This entry updated before 7 March 2001.
22. Section 2.5.11, GENERALIZED INVERSES, page 129, item number 2, the statement is mostly incorrect. Presently we have

Only if $\mathbf{A}$ is square and non-singular, $\mathbf{A}^{+}$will be unique and $\mathbf{A}^{+}=\mathbf{A}^{-1}$. Otherwise, there will exist infinitely many matrices $\mathbf{A}^{+}$that will satisfy the defining relation.

This is incorrect. It should have been
There is a unique pseudoinverse satisfying the conditions in (2.5.26).
If, and only if, $\mathbf{A}$ is square and non-singular, then $\mathbf{A}^{+}=\mathbf{A}^{-1}$.
(Thanks to Pablo A. Parrilo for this correction.)
This entry updated before 7 March 2001.
23. Section 2.5.17, THEOREMS, page 135, equation (2.5.31) presently has (in part)

$$
\mathbf{B}_{j}=\left[\begin{array}{cccc}
a_{11} & a_{12} & b_{1} & a_{1 n} \\
\vdots & \vdots & \vdots & \vdots \\
a_{n 1} & a_{n 2} & b_{n} & a_{n n}
\end{array}\right]
$$

There are missing terms and missing ellipses, this should have been

$$
\mathbf{B}_{j}=\left[\begin{array}{cccccccc}
a_{11} & a_{12} & \ldots & a_{1, j-1} & b_{1} & a_{1, j-1} & \ldots & a_{1 n} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
a_{n 1} & a_{n 2} & \ldots & a_{n, j-1} & b_{n} & a_{1, j-1} & \ldots & a_{n n}
\end{array}\right]
$$

(Thanks to Paul Stanford for this correction.)
This entry updated before 7 March 2001.
24. Section 2.6.8, PERMUTATION GROUPS, pages 147-148, the definition of the cycle index is missing. The following should be included:

Suppose that $\pi$ is a permutation with $b_{1}$ cycles of length $1, b_{2}$ cycles of length 2 , $\ldots, b_{k}$ cycles of length $k$ in its unique cycle decomposition. Then $\pi$ can be encoded as the expression $x_{1}^{b_{1}} x_{2}^{b_{2}} \cdots x_{k}^{b_{k}}$. Summing these expressions for all permutations in the group $G$, and normalizing by the number of elements in $G$ results in the cycle index of the group $G$ :

$$
P_{G}\left(x_{1}, x_{2}, \ldots, x_{l}\right)=\frac{1}{|G|} \sum_{\pi \in G}\left(x_{1}^{b_{1}} x_{2}^{b_{2}} \cdots x_{k}^{b_{k}}\right)
$$

This entry updated before 7 March 2001.
25. Section 3.2.3, BINOMIAL COEFFICIENTS, page 169, the following text should be added

- If $n$ and $m$ are integers, and $m>n$, then $\binom{n}{m}=0$

This entry updated before 7 March 2001.
26. Section 3.2.3, Pascal's triangle, page 170, the table now begins as

\[

\]

which is incorrect, it should have been

> | 1 |
| :--- |
| $\begin{array}{c}111 \\ 121 \\ 1331\end{array}$ |

(Thanks to Katherine Jane Harine for this correction.)
This entry last updated 25 Februrary 2002.
27. Section 3.2.9, STIRLING CYCLE NUMBERS, page 174, the displayed expression is

$$
\left\{\begin{array}{c}
n \\
m
\end{array}\right\}=\frac{1}{k!} \sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i} i^{n}
$$

This is incorrect. It should say (note $k \rightarrow m$ )

$$
\left\{\begin{array}{c}
n \\
m
\end{array}\right\}=\frac{1}{m!} \sum_{i=0}^{m}(-1)^{m-i}\binom{m}{i} i^{n}
$$

There are also several other small errors in sections 3.2.8 and 3.2.9.
(Thanks to David Jeffrey for pointing out many errors.)
This entry last updated 5 October 2001.
28. Section 3.5.7, LATIN SQUARES, page 214, the $n=10$ Latin square has as its fourth row

$$
\begin{array}{llllllll|ll}
94 & 86 & 71 & 33 & 27 & 18 & 09 & 45 & 59 & 62
\end{array}
$$

This is incorrect. This row should have been (look at second to last element)

$$
\begin{array}{llllllllll}
94 & 86 & 71 & 33 & 27 & 18 & 09 & 45 & 50 & 62
\end{array}
$$

This entry updated before 7 March 2001.
29. Section 3.7.4, BINARY SEQUENCES, page 225, table at the top of the page
(a) The sequence marked with a " 7 S " in the first column should be marked by a " 7 ".
(b) The third sequence marked by a " 7 " in the first column should be marked by a " 7 S ".
(Thanks to Joe Rushanan for these corrections.)
This entry updated before 7 March 2001.
30. Section 4.5.13, REGULAR POLYGONS, page 276, in the first set of displayed equations:
(a) the expression "area $=\frac{1}{4} k s^{2} \ldots$ " should have been "area $=\frac{1}{4} k a^{2} \ldots$ "
(b) the expression " $r=\frac{1}{2} s \ldots$ " should have been " $r=\frac{1}{2} a \ldots$.
(c) the expression " $R=\frac{1}{2} s \ldots$ " should have been " $R=\frac{1}{2} a \ldots$ "
(Thanks to L. W. Maxwell for this correction.)
This entry updated before 7 March 2001.
31. Section 4.6, CIRCLES, page 277, just before the last sentence add on the page, add the text (All angles are measured in radians.)

This entry updated before 7 March 2001.
32. Section 4.7.1, ALTERNATIVE CHARACTERIZATION, page 281, last line, the eccentricity is currently written as

$$
\sqrt{a^{2}+b^{2}} / a
$$

This is incorrect. It should have been

$$
\sqrt{a^{2}-b^{2}} / a
$$

(Thanks to Gary E. Young for this correction.)
This entry updated before 7 March 2001.
33. Section 4.7.3, ADDITIONAL PROPERTIES OF ELLIPSES, page 285, at the end of note number 1, the following text should be added:

Note the approximation $C \approx 2 a\left[2+(\pi-2)\left(\frac{b}{a}\right)^{1.456}\right]$
(Thanks to David F. Rivera for this addition.)
This entry updated before 7 March 2001.
34. Section 4.7.3, ADDITIONAL PROPERTIES OF ELLIPSES, page 285
(a) First displayed equation, below and to the left of the figure, the rightmost expression is presently

$$
a \int_{0}^{\theta} \sqrt{1-e^{2} \cos ^{2} \phi} d \phi=a E(\pi / 2-\theta e),
$$

This is incorrect, it should have been

$$
a \int_{0}^{\theta} \sqrt{1-e^{2} \cos ^{2} \phi} d \phi=a\left(E\left(\frac{\pi}{2}, e\right)-E\left(\frac{\pi}{2}-\theta, e\right)\right)
$$

(b) On the next line we present have

Setting $\theta=2 \pi$
This is incorrect, it should have been
Setting $\theta=\pi / 2$
(c) In the next display, the rightmost expression is presently

$$
C=4 a E(0, e) .
$$

This is incorrect, it should have been

$$
C=4 a E(\pi / 2, e) .
$$

(Thanks to David Cantrell for these corrections.)
This entry updated before 7 March 2001.
35. Section 4.8.2, ROULETTES (SPIROGRAPH CURVES) page 292, the second displayed equation is now

$$
x=\cos ^{-1} a-\frac{y}{a} \pm \sqrt{2 a y-y^{2}}
$$

This is incorrect. It should say (note $k \rightarrow m$ )

$$
x= \pm\left(a \cos ^{-1}\left(\frac{a-y}{a}\right)-\sqrt{2 a y-y^{2}}\right)
$$

(Thanks to James Seed for this correction.)
This entry updated before 7 March 2001.
36. Section 4.8.3, SPIRALS, page 294, the first full sentence is now

A curve parameterized by arclength and such that the curvature is proportional to the parameter at each point is a Bernoulli spiral.

This is incorrect. It should say
A curve parameterized by arclength and such that the radius of curvature is proportional to the parameter at each point is a Bernoulli spiral.

This entry updated before 7 March 2001.
37. Section 4.8.3, SPIRALS, page 294, the last sentence is now

A curve parameterized by arclength and such that the curvature is inversely proportional to the parameter at each point is a Cornu spiral (compare the Bernoulli spiral).

This is incorrect. It should say
A curve parameterized by arclength and such that the radius of curvature is inversely proportional to the parameter at each point is a Cornu spiral (compare the Bernoulli spiral).

This entry updated before 7 March 2001.
38. Section 4.10, SPACE SYMMETRIES OR ISOMETRIES, page 300,
(a) Line 5 now has "in a line $P$ ", which is incorrect. It should have been "in a plane $P$ ".
(b) Line 6 now has "in a line $P$ ", which is incorrect. It should have been "in a plane $P$ ".
(c) An additional line, (number 7), should be added. It should say

A rotation-reflection (rotation through an angle $\alpha$ around a line $L$ composed with reflection in a plane perpendicular to $L$ ).
(Thanks to Jerry Grossman for these corrections.)
This entry updated before 7 March 2001.
39. Section 4.15.1, REGULAR POLYHEDRA, page 311, the $V / a^{3}$ ratio for the Dodecahedron is now listed as

$$
\frac{1}{4} \sqrt{15+7 \sqrt{5}} \quad 7.663119
$$

This is incorrect. It should have been

$$
\frac{1}{4}(15+7 \sqrt{5}) \quad 7.663119
$$

(Thanks to David G. Simpson for this correction.)
This entry updated before 7 March 2001.
40. Section 4.18, SPHERES, page 314, line 9 , now has

Four points not on the same line ...
This is incorrect. It should have been
Four points not on the same plane ...
This entry updated before 7 March 2001.
41. Section 4.18, SPHERES, page 315, equation 4.18.2, now has

$$
\begin{aligned}
& V_{n}(r)=\frac{2 \pi r^{2}}{n} V_{n-2}=\frac{2 \pi^{n / 2} r^{n}}{n \Gamma\left(\frac{n}{2}\right)} \\
& S_{n}(r)=\frac{n}{r} V_{n}=\frac{d}{d r}\left[V_{n}(r)\right]
\end{aligned}
$$

This is correct, but could be clarified by writing it as:

$$
\begin{aligned}
& V_{n}(r)=\frac{2 \pi r^{2}}{n} V_{n-2}(r)=\frac{2 \pi^{n / 2} r^{n}}{n \Gamma\left(\frac{n}{2}\right)}=\frac{\pi^{n / 2} r^{n}}{\left(\frac{n}{2}\right)!} \\
& S_{n}(r)=\frac{n}{r} V_{n}(r)=\frac{d}{d r}\left[V_{n}(r)\right]
\end{aligned}
$$

(Thanks to Randolph J. Herber for this clarification.)
This entry updated before 7 March 2001.
42. Section 5.1, DIFFERENTIAL CALCULUS, page 335, last formulae on the page is now

$$
\frac{d y}{d x}=\frac{\dot{y}(t)}{\dot{x}(t)}, \quad \quad \frac{d^{2} y}{d x^{2}}=\frac{\dot{x} \ddot{y}-\ddot{x} \dot{y}}{(\dot{x})^{2}}
$$

This is incorrect. It should have been

$$
\frac{d y}{d x}=\frac{\dot{y}(t)}{\dot{x}(t)}, \quad \quad \frac{d^{2} y}{d x^{2}}=\frac{\dot{x} \ddot{y}-\ddot{x} \dot{y}}{(\dot{x}) 3}
$$

(Thanks to Richard W. Johnson for this correction.)
This entry last updated 20 September 2001.
43. Section 5.1.1, MAXIMA AND MINIMA OF FUNCTIONS, page 338, lines 4-6 are now

$$
2 x+2 \lambda(x-1)=0 \quad 2 y+2 \lambda(y-2)=0 \quad(x-1)^{2}+(y-2)^{2}=0
$$

The solutions are: $\{x=1+1 \sqrt{5}, y=2+2 / \sqrt{5}, \lambda=-1-\sqrt{5}\}$ (furthest), and $\{x=1-1 \sqrt{5}, y=2-2 / \sqrt{5}, \lambda=\sqrt{5}-1\}$ (closest).

These are incorrect. They should have been (note the last equation on the first line is missing a " 1 ", and a slash is missing in the two $x$ solutions):

$$
2 x+2 \lambda(x-1)=0 \quad 2 y+2 \lambda(y-2)=0 \quad(x-1)^{2}+(y-2)^{2}=1
$$

The solutions are: $\{x=1+1 / \sqrt{5}, y=2+2 / \sqrt{5}, \lambda=-1-\sqrt{5}\}$ (furthest), and $\{x=1-1 / \sqrt{5}, y=2-2 / \sqrt{5}, \lambda=\sqrt{5}-1\}$ (closest).
(Thanks to Bruno Van der Bossche for these corrections.)
This entry updated before 7 March 2001.
44. Section 5.1.3, PROPERTIES, page 340, item number 4 now has

$$
\frac{\partial \mathbf{z}}{\partial \mathbf{y}}=\frac{\partial \mathbf{y}}{\partial \mathbf{x}} \frac{\partial \mathbf{z}}{\partial \mathbf{y}}
$$

This is incorrect. It should have been

$$
\frac{\partial \mathbf{z}}{\partial \mathbf{y}}=\frac{\partial \mathbf{z}}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{y}}
$$

(Thanks to Pablo A. Parrilo for this correction.)
This entry updated before 7 March 2001.
45. Section 5.3.5, Useful transformations, page 349, items 4 and 5 now have

$$
\text { when } z=\tan \frac{x}{a}
$$

This is incorrect. It should have been

$$
\text { when } z=\tan \frac{x}{2}
$$

This entry last updated 25 Februrary 2002.
46. Section 5.3.13, ASYMPTOTIC INTEGRAL EVALUATION, page 357, last line on the page, we presently have

$$
\begin{equation*}
\sim g\left(x_{0}\right) \sqrt{\frac{2 \pi}{\lambda\left|f^{\prime \prime}\left(x_{0}\right)\right|}}\left[i \lambda f\left(x_{0}\right)-\frac{i \pi}{4} \operatorname{signum} f^{\prime \prime}\left(x_{0}\right)\right]+\ldots \tag{5.3.21}
\end{equation*}
$$

This is incorrect, it should have been (note that the exponential function is missing, and that the last sign is incorrect)

$$
\begin{equation*}
\sim g\left(x_{0}\right) \sqrt{\frac{2 \pi}{\lambda\left|f^{\prime \prime}\left(x_{0}\right)\right|}} \exp \left[i \lambda f\left(x_{0}\right)+\frac{i \pi}{4} \text { signum } f^{\prime \prime}\left(x_{0}\right)\right]+\ldots \tag{5.3.21}
\end{equation*}
$$

(Thanks to Richard B. Evans for these corrections.)
This entry updated before 7 March 2001.
47. Section 5.3.14, MOMENTS OF INERTIAL FOR VARIOUS BODIES, page 358, entry number 12 which is for a "spherical shell, very thin, mean radius $r$ " the moment of inertia is listed as

$$
m \frac{2}{3} r 3
$$

This is incorrect, it should have been

$$
m \frac{2}{3} r 2
$$

(Thanks to Richard Finley for this correction.)
This entry updated before 7 March 2001.
48. Section 5.4.8, FORMS CONTAINING $a+b x+c x^{2}$, page 367, the second line is now If $q=0$, then $X=c\left(x+\frac{b}{2 c}\right)$ and other formulae should be used.

This is incorrect, it should have been

$$
\text { If } q=0 \text {, then } X=c\left(x+\frac{b}{2 c}\right)^{2} \text { and other formulae should be used. }
$$

This entry last updated 25 Februrary 2002.
49. Section 5.4.11, FORMS CONTAINING $\sqrt{x^{2} \pm a^{2}}$, pages 370-371, integrals number 149, 150,161 , and 164 all have the expression

$$
\log x+\sqrt{x^{2} \pm a^{2}}
$$

This is incorrect, it should have been

$$
\log \left(x+\sqrt{x^{2} \pm a^{2}}\right)
$$

(Thanks to Jonathan Thomas Bartley for these corrections.)
This entry updated before 7 March 2001.
50. Section 5.4.12, FORMS CONTAINING $\sqrt{a^{2}-x^{2}}$, page 373, integral number 208 is missing the $d x$
(Thanks to Guizhong Zhang for this correction.)
This entry updated before 7 March 2001.

## 51. Section 5.5, TABLE OF DEFINITE INTEGRALS,

(a) page 396 , integral number 586 is presently:

$$
\int_{0}^{1} x^{m-1}(1-x)^{n-1} d x=\int_{0}^{1} \frac{x^{m-1}}{(1+x)^{m+n}} d x=\frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)} \quad n>0, m>0 .
$$

This is incorrect and should be (note the upper limit on the second integral):

$$
\int_{0}^{1} x^{m-1}(1-x)^{n-1} d x=\int_{0}^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} d x=\frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)} \quad n>0, m>0 .
$$

(b) page 397, integral number 598 is presently:

$$
\int_{0}^{a}\left(a^{2}-x^{2}\right)^{n / 2} d x=\int_{0}^{a} \frac{1}{2}\left(a^{2}-x^{2}\right)^{n / 2} d x=\frac{n!!}{(n+1)!!} \frac{\pi}{2} a^{n+1}, \quad a>0, n \text { is }
$$

an odd integer.

This is incorrect and should be (note the lower limit on the second integral):

$$
\int_{0}^{a}\left(a^{2}-x^{2}\right)^{n / 2} d x=\int_{-a}^{a} \frac{1}{2}\left(a^{2}-x^{2}\right)^{n / 2} d x=\frac{n!!}{(n+1)!!} \frac{\pi}{2} a^{n+1}, \quad a>0, n \text { is }
$$ an odd integer.

(c) page 398, integral number 606 is presently:

$$
\int_{0}^{\pi / n} \sin n x \cdot \cos n x d x=\int_{0}^{\pi / n} \sin n x \cdot \cos n x d x=0, \quad n \text { is an integer. }
$$

This is correct but should have been (note the upper limit on the second integral):

$$
\int_{0}^{\pi / n} \sin n x \cdot \cos n x d x=\int_{0}^{\pi} \sin n x \cdot \cos n x d x=0, \quad n \text { is an integer. }
$$

FOOTNOTE: An alert reader will wonder how these errors could have occurred, since the integrals in the 30th edition of this book have been electronically verified. The error occurred in the typesetting of the integrals - not in the electronic verification of the integrals. In this section on definite integrals, there are 11 integrals in which there were two integrals on the same line; in all cases the limits on the first integral were (sometimes incorrectly) printed as the limits on the second integral. This resulted in the errors listed above.
(Thanks to Michael Strauss for this correction.)
This entry updated before 7 March 2001.
52. Section 5.5, TABLE OF DEFINITE INTEGRALS, page 400, integral number 646 presently has " $a>0$ ". This statement is not needed since $a$ does not appear in the integral.
(Thanks to Luke Sweatlock for this correction.)
This entry last updated 25 Februrary 2002.
53. Section 5.5, TABLE OF DEFINITE INTEGRALS, page 400, integral number 648 presently has " $n>0$ ". This is incorrect, it should have been " $n>0$ and $n$ even".
(Thanks to Luke Sweatlock for this correction.)
This entry last updated 25 Februrary 2002.
54. Section 5.7.8, SEPARATION OF VARIABLES, on page 423,
(a) line 6 , now has

$$
W=X_{1}\left(u^{1}\right) X_{2}\left(u_{2}\right) X_{3}\left(u^{3}\right)
$$

This is incorrect. It should have been

$$
W=X_{1}\left(u^{1}\right) X_{2}\left(u^{2}\right) X_{3}\left(u^{3}\right)
$$

(b) line 6 , fourth displayed equation is now

$$
S=\left[\begin{array}{ccc}
\mu^{2} & -1 & 1 / \mu^{2} \\
\nu^{2} & 1 & 1 / \nu^{2} \\
0 & 0 & 1
\end{array}\right],
$$

This is incorrect. It should have been

$$
S=\left[\begin{array}{ccc}
\mu^{2} & -1 & -1 / \mu^{2} \\
\nu^{2} & 1 & -1 / \nu^{2} \\
0 & 0 & 1
\end{array}\right],
$$

(c) line 14, now has

$$
M_{21}=\mu^{-2}+\nu^{-2}
$$

This is incorrect. It should have been (note the subscript)

$$
M_{31}=\mu^{-2}+\nu^{-2}
$$

(d) line 15 , now has

$$
f_{2}=f_{3}=1 .
$$

This is incorrect. It should have been

$$
f_{2}=\nu, \text { and } f_{3}=1
$$

(e) fifth displayed equation is now (in part)

$$
\begin{aligned}
& \cdots+X_{1}\left(\alpha_{1} \mu^{2}-\alpha_{2}+\frac{\alpha_{1}}{\mu^{2}}\right)=0 \\
& \cdots+X_{2}\left(\alpha_{1} \nu^{2}+\alpha_{2}+\frac{\alpha_{1}}{\nu^{2}}\right)=0, \text { and }
\end{aligned}
$$

This is incorrect. It should have been

$$
\begin{aligned}
& \cdots+X_{1}\left(\alpha_{1} \mu^{2}-\alpha_{2}-\frac{\alpha_{3}}{\mu^{2}}\right)=0 \\
& \cdots+X_{2}\left(\alpha_{1} \nu^{2}+\alpha_{2}-\frac{\alpha_{3}}{\nu^{2}}\right)=0, \text { and }
\end{aligned}
$$

This entry updated before 7 March 2001.
55. Section 5.10.3, DIFFERENTIATION OF TENSORS, page 433, equation (5.10.8), we presently have

$$
\begin{aligned}
& \nabla_{k} T^{i_{1} \cdots i_{r}}{ }_{j_{1} \cdots j_{s}}=\partial_{k} T^{i_{1} \cdots i_{r}}{ }_{j_{1} \cdots j_{s}}+\Gamma^{i_{1}}{ }_{\ell k} T^{\ell i_{2} \cdots i_{r}}{ }_{j_{1} \cdots j_{s}}+\cdots \\
& \cdots+\Gamma^{i_{r}}{ }_{\ell k} T^{\ell{ }_{i} \cdots i_{r-1} \ell}{ }_{j_{1} \cdots j_{s}}-\Gamma^{\ell}{ }_{j_{1} k} T^{i_{1} \cdots i_{r}}{ }_{\ell j_{2} \cdots j_{s}} \cdots-\Gamma_{j_{s} k}^{\ell} T^{i_{1} \cdots i_{r}}{ }_{j_{1} \cdots j_{s-1} \ell},
\end{aligned}
$$

This is incorrect. It should have been (notice that the second superscript on the second line has an $\ell$ that should be an $i$ )

$$
\begin{aligned}
& \nabla_{k} T^{i_{1} \cdots i_{r}}{ }_{j_{1} \cdots j_{s}}=\partial_{k} T^{i_{1} \cdots i_{r}}{ }_{j_{1} \cdots j_{s}}+\Gamma^{i_{1}}{ }_{\ell k} T^{\ell i_{2} \cdots i_{r}}{ }_{j_{1} \cdots j_{s}}+\cdots \\
& \cdots+\Gamma^{i_{r}}{ }_{\ell k} T^{i_{1} \cdots i_{r-1} \ell}{ }_{j_{1} \cdots j_{s}}-\Gamma^{\ell}{ }_{j_{1} k} T^{i_{1} \cdots i_{r}}{ }_{\ell j_{2} \cdots j_{s}} \cdots-\Gamma_{j_{s} k}^{\ell} T^{i_{1} \cdots i_{r}}{ }_{j_{1} \cdots j_{s-1} \ell},
\end{aligned},
$$

(Thanks to Harry Watson of the Naval Warfare Assessment Division (NWAD) for this correction.)
This entry updated before 7 March 2001.
56. Section 5.11.1, LIST OF ORTHGONAL COORDINATE SYSTEMS, pages 442-445, each formula of the form " $h_{i}=\ldots$ " should have instead been " $f_{i}=\ldots$ ".

This entry updated before 7 March 2001.
57. Section 5.12, CONTROL THEORY, page 445, the derivative on the $\mathbf{y}$ term in the second equation is incorrect, the definition of controllability is incorrect, and the controllability and observability criteria should be be stated as "if and only if". Also, the statements could be made in more generality. The first three paragraphs of this section should be replaced by:

Let $\mathbf{x}$ be a state vector, let $\mathbf{y}$ be an observable vector, and let $\mathbf{u}$ be the control. The vectors $\mathbf{x}, \mathbf{y}$, and $\mathbf{u}$ have $n, m$ and $p$ components, respectively. If a system evolves as:

$$
\begin{aligned}
& \dot{\mathbf{x}}=A \mathbf{x}+B \mathbf{u} \\
& \mathbf{y}=C \mathbf{x}+D \mathbf{u}
\end{aligned}
$$

then, taking Laplace transforms, $\tilde{\mathbf{y}}=G(s) \tilde{\mathbf{u}}$ where $G(s)$ is the transfer function given by $G(s)=C(s I-A)^{-1} B+D$.

A system is said to be controllable if and only if for any times $\left\{t_{0}, t_{1}\right\}$ and any states $\left\{\mathbf{x}_{0}, \mathbf{x}_{1}\right\}$ there exists a control $\mathbf{u}(t)$ such that $\mathbf{x}\left(t_{0}\right)=\mathbf{x}_{0}$ and $\mathbf{x}\left(t_{1}\right)=\mathbf{x}_{1}$. The system is controllable if and only if $\operatorname{rank}\left[B A B A^{2} B \ldots A^{n-1} B\right]=n$.

If, given $\mathbf{u}(t)$ and $\mathbf{y}(t)$ on some interval $t_{0}<t<t_{1}$, the value of $\mathbf{x}(t)$ can be deduced on that interval, then the system is said to be observable. Observability is equivalent to the condition $\operatorname{rank}\left[C^{\mathrm{T}} A^{\mathrm{T}} C^{\mathrm{T}} \ldots\left(A^{(n-1)}\right)^{\mathrm{T}} C^{\mathrm{T}}\right]=n$.
(Thanks to Pablo A. Parrilo for this correction.)
This entry updated before 7 March 2001.
58. Section 6.1.9, CIRCULAR FUNCTIONS OF SOME SPECIAL ANGLES, page 455, there are two errors:

- The sine of $120^{\circ}$ is $\sqrt{3} / 2$ (and not $1 / 2$ )
- The sine of $150^{\circ}$ is $1 / 2$ (and not $\sqrt{3} / 2$ )
(Thanks to Andre D. Bandrauk for these corrections.)
This entry updated before 7 March 2001.

59. Section 6.1.14, DOUBLE ANGLE FORMULAE, page 458, in the sine double angle formula (the first formula), the $\tan ^{2} \alpha$ term in the denominator has an incorrect factor of 2 . It now has:

$$
\sin 2 \alpha=2 \sin \alpha \cos \alpha=\frac{2 \tan \alpha}{1+\sqrt{2} \tan ^{2} \alpha}
$$

It should have been

$$
\sin 2 \alpha=2 \sin \alpha \cos \alpha=\frac{2 \tan \alpha}{1+\tan ^{2} \alpha}
$$

(Thanks to Richard F. Stein for this correction.)
This entry updated before 7 March 2001.
60. Section 6.2.2, GENERAL PLANE TRIANGLES page 461, "Triangle sides in terms of other components", presently has

$$
\begin{aligned}
a & =b \cos C+c \cos B, \\
c & =a \cos A+b \cos C, \\
b & =c \cos B+a \cos A .
\end{aligned}
$$

This is incorect, it should have been

$$
\begin{aligned}
a & =b \cos C+c \cos B, \\
c & =b \cos A+a \cos B, \\
b & =a \cos C+c \cos A .
\end{aligned}
$$

(Thanks to Richard Hughes for this correction.)
This entry last updated 25 Februrary 2002.
61. Section 6.4, SPHERICAL SPHERICAL TRIANGLES page 468. Figure 6.4.3 and the first line of "Napier's rules of cicrular parts" both presently have "co- $C$ ". This is incorrect,it should have been "co-c".
(Thanks to Donal M. Ragan for this correction.)
This entry last updated 25 Februrary 2002.
62. Section 6.10.11, TABLE OF SPHERICAL HARMONICS, page 493, the $Y_{33}$ term is presently

$$
Y_{33}=-\frac{1}{4} \sqrt{\frac{\overline{\boxed{105}}_{4 \pi}^{4}}{}} \sin ^{3} \theta e^{3 i \phi},
$$

This is incorrect. It should have been

$$
Y_{33}=-\frac{1}{4} \sqrt{\frac{35}{4 \pi}} \sin ^{3} \theta e^{3 i \phi},
$$

(Thanks to William Weintraub for this correction.)
This entry last updated 20 September 2001.
63. Section 6.17.3, POLYNOMIAL CASE, page 510, now has (in part)

$$
\sum_{k=0}^{m} \frac{(-1)^{k}(2 n-2 k)!}{2^{k} k!(n-k)!(n-2 k)!} x^{n-2 k},
$$

This is incorrect. It should have been (notice that $2^{k}$ is replaced by $2^{n}$ ):

$$
\sum_{k=0}^{m} \frac{(-1)^{k}(2 n-2 k)!}{2^{n} k!(n-k)!(n-2 k)!} x^{n-2 k},
$$

(Thanks to Michael E. Kutz for this correction.)
This entry updated before 7 March 2001.
64. Section 6.19.3, COMPLEMENTARY INTEGRALS, page 523, now has

$$
K^{\prime}=F\left(k^{\prime}, \frac{1}{2} \pi\right), \quad E^{\prime}=E\left(k^{\prime}, \frac{1}{2} \pi\right), \quad k^{\prime}=\sqrt{1-k^{2}}
$$

This is incorrect. It should have been (notice the order of the arguments)

$$
K^{\prime}=F\left(\frac{1}{2} \pi, k^{\prime}\right), \quad E^{\prime}=E\left(\frac{1}{2} \pi, k^{\prime}\right), \quad k^{\prime}=\sqrt{1-k^{2}}
$$

(Thanks to David W. Cantrell for this correction.)
This entry updated before 7 March 2001.
65. Section 6.30, TABLES OF TRANSFORMS, page 559, Laplace transforms: functional relations, number 12, now has

$$
e^{-b t} F(s)
$$

This is incorrect. It should have been

$$
e^{-b s} F(s)
$$

(Thanks to David Lassonde for this correction.)
This entry updated before 7 March 2001.
66. Section 6.30, TABLES OF TRANSFORMS, page 563, Laplace transforms, number 74, now has

| 74 | $e^{-a \sqrt{t}} \quad(a>0)$ | $\frac{a}{2 \sqrt{\pi \boxed{s^{3}}}} e^{-a^{2} / 4 s}$ |
| :--- | :--- | :--- |

This is incorrect. It should have been

| 74 | $e^{-a \sqrt{t}}$ | $(a>0)$ |
| :--- | :--- | :--- |

(Thanks to Dale Hinds for this correction.)
This entry last updated 20 September 2001.
67. Section 7.1.8, GEOMETRIC PROBABILITY, page 579,

- number 2 , the first displayed equation is now

$$
\begin{array}{r}
\left.P_{k}(x)=\text { Probability (exactly } k \text { intervals have length larger than } x\right) \\
=\binom{n}{k}\left\{[1-k x]^{n-1}-\binom{n-1}{1}[1-(k+1) x]^{n-1}+\right. \\
\left.\cdots+(-1)^{s}\binom{n-k}{s}[1-(k+s) x]^{n-1}\right\}
\end{array}
$$

This is incorrect, it should have been (note that " 1 " should have been a " $k$ " in one of the binomial coefficients)

$$
\begin{aligned}
& P_{k}(x)=\text { Probability (exactly } k \text { intervals have length larger than } x) \\
&=\binom{n}{k}\left\{[1-k x]^{n-1}-\binom{n-k}{1}[1-(k+1) x]^{n-1}+\right. \\
&\left.\cdots+(-1)^{s}\binom{n-k}{s}[1-(k+s) x]^{n-1}\right\}
\end{aligned}
$$

- number 5 (Buffon's needle problem) presently has

A needle of length $L$ is placed at random on a plane on which are ruled parallel lines at unit distance apart. Assume that $L<1$ so that only one intersection is possible. The probability $P$ that the needle intersects a line is

$$
P=\frac{2}{\pi}\left[\frac{\pi}{2}-\arcsin L^{-1}+L-\sqrt{L^{2}-1}\right]
$$

This is incorrect and incomplete (the $\sqrt{L^{2}-1}$ should have been $\sqrt{1-L^{2}}$ ). It should have been:

A needle of length $L$ is placed at random on a plane on which are ruled parallel lines a distance $D$ apart. If $\frac{L}{D}<1$ then only one intersection is possible. The probability $P$ that the needle intersects a line is

$$
P= \begin{cases}\frac{2 L}{\pi D} & \text { if } 0<L \leq D \\ \frac{2 L}{\pi D}\left(1-\sqrt{1-\left(\frac{D}{L}\right)^{2}}\right)+\left(1-\frac{2}{\pi} \arcsin \frac{D}{L}\right) & \text { if } 0<D \leq L\end{cases}
$$

This entry updated before 7 March 2001.
68. Section 7.2.1, DISCRETE DISTRIBUTIONS, page 582, number 5, Negative Binomial Distribution, now has

$$
\mu=\frac{r}{\theta}
$$

and

$$
G(t)=e^{t r} \theta^{r}\left[1-(1-\theta) e^{t}\right]^{-r}
$$

They are both incorrect. They should have been

$$
\mu=\frac{r}{\theta}-r
$$

and

$$
G(t)=\theta^{r}\left[1-(1-\theta) e^{t}\right]^{-r}
$$

(Thanks to Dillard David Ensley for this correction.)
This entry updated before 7 March 2001.
69. Section 7.2.2, CONTINUOUS DISTRIBUTIONS, page 583, note number 2, Normal Distribution, now has
then the variable $X$ is said to possess a uniform distribution
This is incorrect. It should have been
then the variable $X$ is said to possess a normal distribution
(Thanks to Ian M. Dew of Pacific-Sierra Research Corp. for this correction.)
This entry updated before 7 March 2001.
70. Section 7.2.2, CONTINUOUS DISTRIBUTIONS, page 586, part 9, Student's $t$-distribution, equation (7.2.32) presently has

$$
f(x)=\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n \pi} \Gamma\left(\frac{n}{2}\right)\left(1+\frac{t^{2}}{n}\right)^{(n+1) / 2}} \quad \text { for }-\infty<x<\infty
$$

This should have been (note that $t^{2}$ should be $x^{2}$ ):

$$
f(x)=\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n \pi} \Gamma\left(\frac{n}{2}\right)\left(1+\frac{x^{x}}{n}\right)^{(n+1) / 2}} \quad \text { for }-\infty<x<\infty
$$

(Thanks to Paul Stanford for this correction.)
This entry updated before 7 March 2001.
71. Section 7.4.1, TRANSITION FUNCTION, page 589, line 4, we presently have
$P(x, y)$ is the probability that a Markov chain in state $x$ at time $n$ will be in state $y$ at time $n+1$.

This is incorrect. It should have been
$P(x, y)$ is the probability that a Markov chain in state $x$ at time $t_{n}$ will be in state $y$ at time $t_{n+1}$.
(Thanks to Harry Watson of the Naval Warfare Assessment Division (NWAD) for this correction.)
72. Section 7.4.2, TRANSITION MATRIX, page 590, top of page, we presently have

Define the $n$-step transition matrix by $\mathbf{P}^{(n)}$ as the matrix with entries

$$
P^{n}(x, y)=\ldots
$$

This is incorrect. It should have been
Define the $n$-step transition matrix by $\mathbf{P}^{(n)}$ as the matrix with entries

$$
P^{(n)}(x, y)=\ldots
$$

(Thanks to Harry Watson of the Naval Warfare Assessment Division (NWAD) for this correction.)
This entry updated before 7 March 2001.
73. Section 7.4.4, STATIONARY DISTRIBUTIONS, page 592, the matrix at the top of the page is incorrect. We presently have

$$
P^{(2)}=P^{2}=\left[\begin{array}{ccc}
\frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\
\frac{5}{16} & \frac{9}{16} & \frac{1}{8} \\
\frac{3}{16} & \frac{9}{16} & \frac{1}{8}
\end{array}\right]
$$

This is incorrect, it should have been

$$
P^{(2)}=P^{2}=\left[\begin{array}{ccc}
\frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\
\frac{5}{16} & \frac{9}{16} & \frac{1}{8} \\
\frac{3}{16} & \frac{3}{4} & \frac{1}{16}
\end{array}\right]
$$

(Thanks to Dennis J. Day for this correction.)
This entry updated before 7 March 2001.
74. Section 7.5.1, METHODS OF PSEUDORANDOM NUMBER GENERATION, page 594, part 9 , second table on page, first table after equation (7.5.3) is presently

| Primitive trinomial exponents |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(5,2)$ | $(7,1)$ | $(7,3)$ | $(17,3)$ | $(17,5)$ | $(17,6)$ |
| $(31,3)$ | $(31,6)$ | $(31,7)$ | $(31,13$ | $(127,1)$ | $(521,32)$ |

This should have been (note the missing closing parenthesis for the $31-13$ entry):

| Primitive trinomial exponents |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(5,2)$ | $(7,1)$ | $(7,3)$ | $(17,3)$ | $(17,5)$ | $(17,6)$ |
| $(31,3)$ | $(31,6)$ | $(31,7)$ | $(31,13)$ | $(127,1)$ | $(521,32)$ |

(Thanks to Paul Stanford for this correction.)
This entry updated before 7 March 2001.
75. Section 7.5.2, GENERATING NONUNIFORM RANDOM VARIABLES, page 597, line 4 , presently there is

$$
\mathbf{p}=\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)
$$

This is incorrect. This should have been

$$
\mathbf{p}=\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)
$$

This entry updated before 7 March 2001.
76. Section 7.7.4, ORDER STATISTICS, page 607, we presently have
$\ldots$ let $Z_{m}$ be the $m$ th largest of the values $(m=0,1, \ldots, n)$. Hence $Z_{1}$ is the maximum of the $n$ values and $Z_{n}$ is the minimum of the $n$ values. Then $F_{Z_{m}}(x)=$ $\sum_{m=1}^{n}\binom{n}{i}\left[F_{X}(x)\right]^{i}\left[1-F_{X}(x)\right]^{n-i}$. Hence

$$
\begin{array}{ll}
F_{\min }(z)=\left[F_{X}(z)\right]^{n} & f_{\min }(z)=n\left[F_{X}(z)\right]^{n-1} f_{X}(z) \\
F_{\max }(z)=1-\left[1-F_{X}(z)\right]^{n} & f_{\max }(z)=n\left[1-F_{X}(z)\right]^{n-1} f_{X}(z)
\end{array}
$$

This is incorrect. It should have been (note 3 things: the range of values, the definition for $F_{Z_{m}}(x)$, and the subscripts in the minimum and maximum densities):
$\ldots$ let $Z_{m}$ be the $m$ th largest of the values $(m=1,2, \ldots, n)$. Hence $Z_{1}$ is the maximum of the $n$ values and $Z_{n}$ is the minimum of the $n$ values. Then $F_{Z_{m}}(x)=$
$\sum_{i=m}^{n}\binom{n}{i}\left[F_{X}(x)\right]^{i}\left[1-F_{X}(x)\right]^{n-i}$. Hence

$$
\begin{array}{ll}
F_{\boxed{\max }}(z)=\left[F_{X}(z)\right]^{n} & f_{\boxed{\max }}(z)=n\left[F_{X}(z)\right]^{n-1} f_{X}(z) \\
F_{\overline{\min }}(z)=1-\left[1-F_{X}(z)\right]^{n} & f_{\boxed{\boxed{\min }}}(z)=n\left[1-F_{X}(z)\right]^{n-1} f_{X}(z)
\end{array}
$$

This entry updated before 7 March 2001.
77. Section 7.8.1, CONFIDENCE INTERVAL: SAMPLE FROM ONE POPULATION page 609 , number 3 , bullet 4 , we presently have
... confidence interval for $\hat{p}$ is given by ...
This is incorrect. It should have been
... confidence interval for $p$ is given by ...
This entry updated before 7 March 2001.
78. Section 7.13.3, MATCHED FILTERING (WEINER FILTER), page 660, the name "Weiner" is incorrect; it should have been "Wiener".
(Thanks to Pablo A. Parrilo for this correction.)
This entry updated before 7 March 2001.
79. Section 8.1.4, FITTING EQUATIONS TO DATA, page 682, Best fit line, now has

Given the points $P_{1}\left(x_{1}, y_{1}\right), P_{2}\left(x_{1}, y_{1}\right), \ldots P_{n}\left(x_{1}, y_{1}\right)$ the $\ldots$
This is incorrect. It should have been (notice the subscripts)
Given the points $P_{1}\left(x_{1}, y_{1}\right), P_{2}\left(x_{2}, y_{2}\right), \ldots P_{n}\left(x_{n}, y_{n}\right)$ the $\ldots$
(Thanks to Harry Watson of the Naval Warfare Assessment Division (NWAD) for this correction.)
This entry updated before 7 March 2001.
80. Section 8.2.8, NONLINEAR SYSTEMS AND NUMERICAL OPTIMIZATION, page 691, Method of steepest descent, first displayed equation now ends

$$
\left.\ldots, x_{n}\right]^{2}
$$

This is incorrect. It should have been (i.e., close the parenthesis)

$$
\left.\left.\ldots, x_{n}\right)\right]^{2}
$$

(Thanks to Jeffrey D. Oldham for this correction.)
This entry updated before 7 March 2001.
81. Section 8.3.1, NUMERICAL INTEGRATION, page 693, Closed Newton-Cotes formulae, number 3 , a closing square bracket is missing. That is, " $f\left(x_{3}\right)-$ " should have been " $\left.f\left(x_{3}\right)\right]$ ".
(Thanks to Pablo A. Parrilo for this correction.)
This entry updated before 7 March 2001.
82. Section 8.3.1, NUMERICAL INTEGRATION, page 695, the title Method H.2.1.2 Romberg Integration should have been Romberg Integration.
This entry updated before 7 March 2001.
83. Section 8.3.1, NUMERICAL INTEGRATION, page 695, the following should be added:

## Double integrals of polynomials over polygons

If the vertices of the polygon $A$ are $\left\{\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{p}, y_{p}\right)\right\}$, and we define $w_{i}=x_{i} y_{i+1}-x_{i+1} y_{i}\left(\right.$ with $x_{p+1}=x_{1}$ and $\left.y_{p+1}=y_{1}\right)$ then

$$
\iint_{A} x^{m} y^{n} d A=\frac{m!n!}{(m+n+2)!} \sum_{i=1}^{p} w_{i} \sum_{j=0}^{m} \sum_{k=0}^{n}\binom{j+k}{j}\binom{m+n-j-k}{n-k} x_{i}^{m-j} x_{i+1}^{j} y_{i}^{n-k} y_{i+1}^{k}
$$

(Thanks to Joaquin Marin for this addition.)
This entry updated before 7 March 2001.
84. Section 9.1.3, EXAMPLES page 722,

- Question number 5, answer. The computation as stated is correct, but the numerical evaluation was performed incorrectly. The monthly payment is not $\$ 755.63$, it should be $\$ 804.62$.
- Question number 6. The statement "spending $\$ 600$ per month" should be changed to "spending $\$ 800$ per month".
- Question number 6 , analysis. The statement " $m=600$ " should be changed to " $m=$ 800 ".

This entry updated before 7 March 2001.
85. Section 10.7.1, CONTACT INFORMATION, page 747, note number 7, we presently have

Mathematica http://www.wri.com
While this works, a better reference is
Mathematica http://www.wolfram.com
(Thanks to David Gehrig for this correction.)
This entry updated before 7 March 2001.
86. Section 10.10, ASCII CHARACTER CODES, page 755, the table now has a heading of

American Standard Code for Information Exchange
This is incorrect. It should have been
American Standard Code for Information Interchange
(Thanks to Alex Fabrikant for this correction.)
This entry last updated 5 October 2001.
87. List of Notations, page 757, the following should be added:
$\lfloor\ldots\rfloor \quad$ is the floor function (greatest integer less than or equal to the argument)
$\lceil\ldots\rceil$ is the ceiling function (least integer larger than or equal to than the argument)
(Thanks to David Cantrell for this correction.)
This entry last updated 25 Februrary 2002.

## Dates of updates and errata numbers modified at those dates

2001/3/7
$1,2,4-13,15-17,19-25,28-41,43,44$,
$46,47,49-51,54-59,63-65,67-70,72-$ 85
2001/9/20

42, 62, 66
2001/10/ 5
14, 27, 86
2002/ 2/25
$3,18,26,45,48,52,53,60,61,87$

